STUDIES IN LOGIC.

Studies in logic. By members of the Johns Hopkins university. Boston, *Little, Brown, & Co.*, 1883. 7 + 203 p., 2 pl. 16°.

MR. C. S. PEIRCE and four of his students, present or recent members of his logic classes at Baltimore, offer us in this work six distinct essays on topics of recent logical theory, besides three shorter contributions classed as notes. The volume is throughout studiously unpretentious and very solid work, that might have made much greater claims with perfect safety. The style is extremely compact, and the purchaser of the book will pay for no padding.

Four of the longer studies appeal only to very special students. The two others, Mr. Marquand's essay on the 'Logic of the Epicureans' and Mr. Peirce's very important study of the logic of induction, entitled 'A theory of probable inference,' will interest the general student either of philosophy or of scientific method.

Mr. Marquand's essay on the Epicurean logic opens the book, and gives us an account of the Epicurean theory of induction as it is stated in the work of Philodemus, that has been preserved in fragments in a Herculaneum papyrus. One could wish that this essay had been fuller upon some points; but as a whole we must accept it with thankfulness, as containing useful and not otherwise so easily accessible information. Mr. Marquand then discusses a 'Machine for producing syllogistic variations,' and adds a 'Note on an eight-term logical machine.'

Then follow two 'Algebras of logic,' by Miss Christine Ladd (now Mrs. Fabian Franklin) and Mr. O. H. Mitchell respectively. These are new structures on Boole's foundation. Miss Ladd uses two copulas, expressed by the symbols $\bar{\vee}$ and $\vee$. With these she is able to write algebraically all the old forms of statement, and to perform the customary operations of symbolic logic with great brevity and facility. The copula $\bar{\vee}$, a wedge, is used to signify exclusion. $A \bar{\vee} B$ means that A is wholly excluded from B; i.e., that no A is B. This copula is not to imply the existence of the terms of the statement. The copula $\vee$, an incomplete wedge, is the symbol of imperfect exclusion. $A \vee B$ means that some A is B. And this copula is taken to imply the existence of the terms of the statement. The symbol ∞ is used for the universe of discourse. The symbol 0 finds no use in this algebra. $x \bar{\vee} \infty$ expresses the non-existence of the class x; and this is written more briefly $x \bar{\vee}$. The notation thus established has the convenience that $a \bar{\vee} b = ab \bar{\vee}$, $abc \bar{\vee} = a \bar{\vee} bc$, etc., and, with a corresponding notation for the other copula, $abc \vee = a \vee bc$, etc.; so that the factors of an excluded or not excluded combination may be written in any order, and the copula may be inserted at any point or written at either end. The notation is further applied to combinations of propositions, and to the processes of elimination; and the relative simplicity of expression is preserved throughout.

Mr. Mitchell expresses propositions as logical polynomials, consisting of sums of terms, formed after Boole's fashion. The classes indicated by the polynomials are stated in the propositions to form either the whole or some part of the universe of discourse. Thus, the proposition that the universe $U = \bar{a} + \bar{b}$ would mean that no a is b. Such a proposition Mr. Mitchell expresses by the notation $(\bar{a} + \bar{b})_1$; or, in general, if F be any logical polynomial, F_1 means that F precisely fills up the universe. F_u would express that F forms some part of the universe. $\bar{F}_u$ means that $\bar{F}$ forms part of the universe. Propositions thus formed are used for the purposes of inference in a simple way, expressed in Mr. Mitchell's words by the rule, "Take the logical product of the premises, and erase the terms to be eliminated."

The foregoing may serve to suggest to any one acquainted with Boole's notation the drift of the innovations proposed in these two algebras. Psychological importance, as Mr. Peirce himself suggests, these two notations can hardly claim. They tell us nothing new about the nature of the thinking process, but are interesting only as ingenious and possibly useful methods for expressing very briefly complex facts and elaborate logical calculations. As such expressions, they will hold their own, and may even be noticed in that not very distant time when the whole earth shall be filled with logical algebras, whereof there shall be, for all we can now see, as many as there are tiles on the roofs of the houses.

Mr. B. I. Gilman's very special study follows, on 'Operations in relative number, with application to the theory of probabilities.' Then comes the strong piece of the book, Mr. Peirce's before-mentioned discussion of the logic of induction. This we have read, not with entire conviction, but certainly with no little admiration. Readers of Mr. Peirce's fine papers called 'Illustrations of the logic of science,' in the *Popular science monthly* of

some years back, will be glad to find here, in a more elaborate and technical form, the theory of induction that was outlined in one of those papers. It is, philosophically considered, the most ingenious account of the subject that we have anywhere read; but, as said, we still hesitate to accept this account as complete. But space forbids any lengthy statement of our difficulties in this connection. We must be content with few words.

Mr. Peirce brings the theory of induction into direct connection with the general theory of probable inference, but does so in a way of his own. He rejects, in the first place, any notion that the occurrence or non-occurrence of an event in the past in any way affects the probability of its occurrence in the future. The doctrine of inverse probabilities, as it has hitherto been applied, Mr. Peirce considers as furnishing no foundation for the theory of induction, and equally does he reject our old and trusted friend, the postulate of the uniformity of nature, as the basis of inductive inference. One may well ask, remembering Hume, what yet remains when these faithful allies have failed. But Mr. Peirce's insight finds yet another resource, — not the probability that a given event will be repeated in the future, but the probability that a given form of inference would, in any constitution of the universe whatever, tend in the long-run to lead us to truth rather than to error: this is, for Mr. Peirce, the ground of the true inductive inference. Thus, then, the universe need have no peculiar constitution to render inductive inference valid.

The inductive inference, then, is to be expressed as one form of probable inference. Simple Probable Deduction is exemplified in the typical syllogism:

The proportion ρ of the M's are P's;
S is an M;
It follows, with a probability ρ, that S is a P.

This means that the conclusion, S is P, would in the long-run, and if S is chosen at random, be true in a proportion, ρ, of cases. — More complex is Statistical Deduction, of the form:

The proportion r of the M's are P's;
S′, S″, S‴ are a *numerous* set, taken at random from among the M's:
Hence, probably and approximately, the proportion r of the S's are P's;

that is, the more M's we choose at random, the more likely it is that the same proportion of P's will appear among the chosen M's as exists among the whole actual number of M's. — But now suppose, that, knowing nothing of the real proportion of P's among the M's, we undertake to discover this proportion by sampling the M's. Then we have but to employ our previous principle, and say that the more M's we choose at random, the more will it be likely that the proportion of P's among the chosen M's will equal, and so will reveal, the actual proportion of the P's among all the M's. But now we have induction. We do not assume any thing about the constitution of the unknown parts of the class M. We make no postulate of the 'uniformity' of the class M. That I have found one M that is P, or more, makes it no more probable that the next M found will be P. But we conclude only that the conclusion reached in the following syllogism is reached by a method or precept that must in the long-run lead us towards truth, and away from error. The typical inductive syllogism is:

S′, S″, S‴, etc., form a numerous set, taken at random from among the M's;
S′, S″, S‴, etc., are found to be — the proportion ρ of them — P's:
Hence, probably and approximately, the same proportion, ρ, of the M's are P's.

Thus sampling, continued and fair, tends toward truth, and gives us justifiable ampliative inferences, whatever the constitution of the things about which we infer. Mr. Peirce applies a similar analysis to the form of induction which he calls hypothesis.

This is a very inadequate sketch of a view that deserves serious attention. Of all attempts at a purely empirical theory of our knowledge of nature, this is one of the most promising. We should be sorry to prejudge it in any way by adding to our lame exposition hasty criticism; but, when we say that the theory seems to us to fail just at the most important point, we express what, fairly or unfairly, many readers will feel. The most important point lies in the words 'chosen at random.' Mr. Peirce himself, with perfect fairness, suggests some of the difficulties involved in this word. 'Sampling,' he says 'is a real art, well deserving an extended study by itself.' But does not this art depend for its very existence on an *a priori* assumption about the structure of the universe? Is not a world of which we know that in it we can choose our S's at random from among the M's a world of which we already must know a good deal? Mr. Peirce makes one admission about such a world. It is, he tells us, a world in which we must assume that there are no supernatural and malignant powers at work confusing our choice; i.e., making our supposed random

choice really unfairly predetermined and so deceptive. If, he thinks, the supernatural powers let us alone to choose for ourselves, then our inductions, properly guarded, will inevitably lead us in the direction of true conclusions, whatever the arrangement of the real world. But has Mr. Peirce made all the necessary admissions? Would a devil be needed to confuse my efforts at sampling, so as to make my choice unfair? Would not an instinctive interest in one class of cases serve to vitiate the fairness of my observations in cases where this instinct controlled me? Suppose that by instinct I took such interest in the cases of M's that are P that I noticed no cases, or very few cases, of M's that are not P, however many there might actually be: then, unless I were conscious of this instinctive preference, I should go on neglecting numberless cases that I ought to have taken into account in forming my induction; and yet, not knowing my own natural defect, I should think that I was choosing my cases wholly at random. Here would be a constant error in the process, whose magnitude might be enormous. Yet the error could never be discovered, save by some one to whom a new mental growth made possible the discovery of the instinct. But this case is no factitious one. Our observation of nature is doubtless determined throughout by our natural interests in things. These interests are instinctive, and they may exclude from the very possibility of notice very many facts. Thus, a person that by nature is indisposed to notice the double images in the binocular visual field will study his field of vision for a long time, and will assure you that there is no doubleness there. Might he not say, that after making at random many trials, and finding no double images, he was warranted in the conclusion that for him the proportion of double images in the visual field must be extremely small? Yet once begin to notice the doubleness, and the double images will be found in multitudes, like the chariots and horses that Elisha's servant saw when his eyes were 'opened.'

When we conclude that continuous random sampling of a given natural class must lead us towards discovering the true proportion of cases of the presence of a predesignated character in individuals of the class, must we not base our conclusion on the ultimate *a priori* assumption that our instinctive tendencies to observe natural facts are such as, in the long-run, will lead us to actual choice at random, and not to a choice unconsciously vitiated by unknown preferences for cases that favor the conclusion that we reach? And is not induction, therefore, still dependent on an *a priori* assumption about the nature of reality? C. S. P.

But these inadequate negative suggestions must not give the impression that the foregoing is the whole substance of this very compact essay, which is full of valuable thoughts upon scientific method, and which must be read in detail to be appreciated. We hope for much more such work as this book contains, for the result cannot fail to be of value alike to American science and to American philosophy. Those who oppose a purely empirical philosophy must still be aided by finding so able a defence of some of its doctrines, and those who believe in other forms of logical doctrine cannot afford to remain ignorant of the advances of symbolic logic.

THE RACES OF MEN.

Les races humaines. Par Abel Hovelacque, professeur à l'École d'anthropologie. Paris, *Cerf*, 1882. 159 p., illustr. 16°.

This rather attractive work is written on a practical plan, which is specially useful in tending to correct the false impressions generally entertained, connected with the term 'race.' It is strictly limited to ethnography as distinguished from ethnogeny and ethnology, and simply considers the actual divisions of mankind, with their geographical areas, and their physical, intellectual, and moral characteristics. In the classification of races, the old division by color — as white, yellow, black, etc. — is repudiated; the fact being established, that other characteristics, such as those relating to the hair, to the shape of the cranium, and to height, are equally important, and that none of them can be exclusively adopted in class arrangement. Failure likewise attends a merely linguistic and a strictly geographical grouping. The attempt to discuss races in the order of their development toward civilization would seem to be philosophic, but meets with the difficulty that bodies of men, who, by all other considerations are to be included in the same race, are at wholly diverse degrees of progress in civilization. Admitting, therefore, that no single criterion is possible, the author decided to take account, with due weight, of all the different elements of classification, and to leave to the presentation itself, by its success, the responsibility of justifying its own order.

Professor Hovelacque's arrangement, as distinguished from strict classification, is as follows: 1. Australians; 2. Papuans; 3. Mela-